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CREEP OF AXISYMMETRICALLY LOADED PLATES WITH ALLOWANCE FOR DAMAGE ACCUMULATION IN THEIR MATERIAL

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The Rabotnov kinetic creep theory was used to calculate the stress–strain state and damage accumulation in the material of axisymmetrically loaded circular and ring plates at any time before the beginning of fracture. It is shown that the solution of the problem can be reduced to solving the same problem under the assumption of steady-state creep of the material. The unsteady creep problem is solved by multiplying the known solution of the steady-state creep problem by certain functions of the coordinates and time, which are determined from a corresponding system of equations.

Key words: *creep, damage parameter, time of onset of fracture, axisymmetrically loaded plates.*

Methods for calculating the stress–strain state in uniformly heated and axisymmetrically loaded circular and ring plates assuming steady-state creep of their material are widely known [1–3]. Below, similar calculations are performed using the Rabotnov kinetic theory, which describes all three stages of creep of the material with allowance for damage accumulation in it from phenomenological positions. This model is described by relations [2, 4]

$$\eta_{kl} = \frac{W}{\sigma_{\rm e}} \frac{\partial \sigma_{\rm e}}{\partial \sigma_{kl}}, \qquad W = \frac{B_1 \sigma_{\rm e}^{n+1}}{\phi(\mu)}, \qquad \phi(\mu) = \mu^m (1-\mu)^{\alpha/(\alpha+1)}, \qquad k = 1,2,3, \qquad l = 1,2,3; \tag{1}$$

$$\frac{d\mu}{dt} = -\frac{(\alpha+1) B_2 \sigma_{*\rm e}^{g+1}}{\mu^m}, \qquad \mu(x_k, 0) = 1, \quad \mu(x_k^*, t_*) = 0, \tag{2}$$

where η_{kl} and σ_{kl} are the components of the creep strain rate and stress tensors, $W = \eta_{kl}\sigma_{kl}$ is the power of dissipated energy, $\sigma_{\rm e}$ and $\sigma_{*\,\rm e}$ are first-order functions which are homogeneous with respect to the stresses (as the equivalent stress $\sigma_{\rm e}$, one can adopt the stress intensity σ_i when employing the Mises criterion or the maximum tangential stress when employing the Tresca criterion, etc.; as the equivalent stress $\sigma_{*\,\rm e}$ one can adopt the stress intensity σ_i when using the Kats long-term strength criterion, or the maximum normal stress when using the Johnson long-term strength criterion, etc.). The parameter μ describes the damage accumulation in materials from phenomenological positions. When the material is intact at all points of a body (structural member) with coordinates x_k ($k = 1, 2, 3$), the parameter μ is equal to unity. If at a time $t = t_*$, the parameter μ reaches the critical value equal to zero at any point with coordinates x_k^*, we assume that fracture occurred at this point of the body. The time $t = t_*$ is called the time of onset of fracture of the body. Let us derive the basic equations for calculating the stress–strain state of axisymmetrically loaded circular and ring plates and the damage accumulated in their material.

1. A plane stress state is assumed to occur in the plate, and tangential stresses in the annular section are ignored. As $\sigma_{\rm e}$ and $\sigma_{*\,\rm e}$, we adopt the stress intensity σ_i. The tangential stress σ_{11} will be denoted by σ_φ, and the radial stress σ_{22} by σ_r. The stress $\sigma_{33} = \sigma_z = 0$. Then, under the assumptions made above, system (1) becomes

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$$\eta_\varphi = \frac{B_1}{2\phi(\mu)}\,\sigma_i^{n-1}(2\sigma_\varphi - \sigma_r), \qquad \eta_r = \frac{B_1}{2\phi(\mu)}\,\sigma_i^{n-1}(2\sigma_r - \sigma_\varphi),$$

$$\eta_z = -\frac{B_1}{2\phi(\mu)}\,\sigma_i^{n-1}(\sigma_\varphi + \sigma_r), \tag{3}$$

where

$$\sigma_i = \sqrt{\sigma_\varphi^2 - \sigma_r\sigma_\varphi + \sigma_r^2}. \tag{4}$$

According to Eqs. (1), $W = B_1\sigma_i^{n+1}/\phi(\mu)$; at the same time, $W = \sigma_i\eta_i$. From this, for the creep strain rate intensity, we obtain

$$\eta_i = B_1\sigma_i^n/\phi(\mu). \tag{5}$$

In view of the incompressibility of the material, η_i is expressed in terms of the creep strain rate tensor components:

$$\eta_i = (2/\sqrt{3}\,)\sqrt{\eta_\varphi^2 + \eta_\varphi\eta_r + \eta_r^2}. \tag{6}$$

From (3), we can express σ_φ and σ_r in terms of η_φ and η_r. In view of (5), we obtain

$$\sigma_\varphi = \frac{4}{3}\,\frac{\eta_i^{(1-n)/n}}{B_1^{1/n}}\Big(\eta_\varphi + \frac{1}{2}\,\eta_r\Big)[\phi(\mu)]^{1/n}, \qquad \sigma_r = \frac{4}{3}\,\frac{\eta_i^{(1-n)/n}}{B_1^{1/n}}\Big(\eta_r + \frac{1}{2}\,\eta_\varphi\Big)[\phi(\mu)]^{1/n}. \tag{7}$$

The deflection of a plate of thickness h bent by an axisymmetrically load will be denoted by w, the rotation angle of the normal to the plate by ν. Then, $dw/dr = -\nu$. If ψ is the rate of rotation of the normal, i.e., $\dot\nu = \psi$, $d\dot w/dr = -\psi$ ($\dot w$ is the rate deflection of the middle plane of the plate). We assume the plate deflections to be small compared to its thickness and use the Kirchhoff–Love hypotheses, which corresponds to the state of pure bending. Then, if u is the displacement of any point of the plate in the radial direction, $u(r) = z\nu(r)$. Taking into account that for axisymmetric deformation, $\eta_r = d\dot u/dr$ and $\eta_\varphi = \dot u/r$, we obtain

$$\eta_r = z\,\frac{d\psi}{dr} = -z\,\frac{d^2\dot w}{dr^2}, \qquad \eta_\varphi = z\,\frac{\psi}{r} = -z\,\frac{1}{r}\,\frac{d\dot w}{dr}. \tag{8}$$

In view of (8), the creep strain rate intensity (6) is written as

$$\eta_i = \frac{2}{\sqrt{3}}\,\sqrt{\Big(\frac{\psi}{r}\Big)^2 + \frac{\psi}{r}\,\frac{d\psi}{dr} + \Big(\frac{d\psi}{dr}\Big)^2}\;|z| = \frac{2}{\sqrt{3}}\,\varkappa|z|. \tag{9}$$

Using (8) and (9), we write the stress components (7) as

$$\sigma_\varphi = [\phi(\mu)]^{1/n}a\varkappa^{(1-n)/n}\alpha_\varphi|z|^{(1-n)/n}z, \qquad \sigma_r = [\phi(\mu)]^{1/n}a\varkappa^{(1-n)/n}\alpha_r|z|^{(1-n)/n}z. \tag{10}$$

Here

$$a = \frac{(2/\sqrt{3}\,)^{(n+1)/n}}{(B_1)^{1/n}}, \qquad \alpha_\varphi = \frac{1}{2}\,\frac{d\psi}{dr} + \frac{\psi}{r}, \qquad \alpha_r = \frac{d\psi}{dr} + \frac{1}{2}\,\frac{\psi}{r}, \qquad \psi = \psi(r,t). \tag{11}$$

Substitution of (10) into (4) yields

$$\sigma_i = (\sqrt{3}/2)[\phi(\mu)]^{1/n}a\varkappa^{1/n}|z|^{1/n}. \tag{12}$$

The intensities of the circumferential and radial bending moments in the radial and circular sections of the plate are linked to the circumferential and radial stresses by the well-known relations [1–3]

$$M_\varphi = \int\limits_{-h/2}^{h/2}\sigma_\varphi z\,dz, \qquad M_r = \int\limits_{-h/2}^{h/2}\sigma_r z\,dz.$$

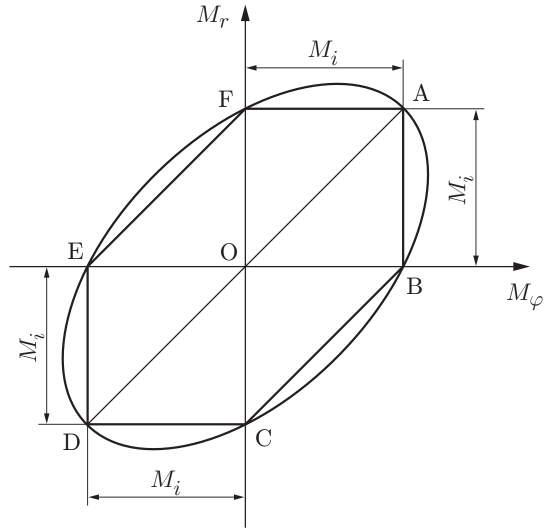

Fig. 1. Relation between the intensities of the bending moments M_r and M_φ.

Relations (10) can be reduced to the form [1, 2]

$$\sigma_\varphi = [\phi(\mu)]^{1/n}(a/D)|z|^{(1-n)/n} z M_\varphi = l M_\varphi, \quad \sigma_r = [\phi(\mu)]^{1/n}(a/D)|z|^{(1-n)/n} z M_r = l M_r, \tag{13}$$

where

$$D(r,t) = 2a \int\limits_0^{h/2} [\phi(\mu)]^{1/n} z^{(n+1)/n}\, dz. \tag{14}$$

The equilibrium equations are written as follows [1–3]:

$$\frac{dM_r}{dr} + \frac{M_r - M_\varphi}{r} = Q, \qquad \frac{d}{dr}\,(Qr) = -pr. \tag{15}$$

Here Q is the intensity of the transverse force in the annular section of the plate and p is the intensity of the uniformly distributed external load; the boundary conditions for $M_r(r)$ on the inner and outer radii are assumed to be specified.

Using (4) and (13), we express the creep strain rate components in relations (1) in terms of the intensities of the bending moments:

$$\eta_\varphi = \Big(\frac{2}{\sqrt{3}}\Big)^{n+1} \frac{z}{D^n}\, M_i^n\, \frac{\partial M_i}{\partial M_\varphi}, \qquad \eta_r = \Big(\frac{2}{\sqrt{3}}\Big)^{n+1} \frac{z}{D^n}\, M_i^n\, \frac{\partial M_i}{\partial M_r}. \tag{16}$$

Here $M_i = \sqrt{M_\varphi^2 - M_\varphi M_r + M_r^2}$ and $\sigma_i = l M_i$.

From system (2)–(16), one can calculate the stress–strain state of axisymmetrically loaded circular and ring plates at any time before the beginning of fracture.

2. The solution of a number of practical problems using the Mises criterion ($\sigma_{\rm e} = \sigma_i$) involves difficulties and is generally possible only using numerical methods, for example, finite element methods. The quadratic Mises criterion can be linearized to simplify the calculations. For a constant M_i, the plot of the relation between the intensities of the bending moments M_r and M_φ is an ellipse whose major axes are inclined at an angle of 45° to the coordinate axes and which cuts segments equal to M_i (Fig. 1) on these axes. This ellipse can be replaced by a hexagon ABCDEF. Then, the quantity M_i can be approximately expressed in terms of the intensities M_r and M_φ [1, 2], which is equivalent to the linearization of the Mises criterion and to the transition to the Tresca criterion in the basic equations (1) and, hence, in (16). For example, for $M_\varphi > M_r > 0$, the point (M_φ, M_r) belongs to the region AOB ($M_i = M_\varphi$); for $M_\varphi > 0$ and $M_r < 0$, the point (M_φ, M_r) belongs to the region BOC ($M_i = M_\varphi - M_r$), etc.

As an example, we consider the case where $M_i = M_\varphi - M_r$. According to (16),

$$\eta_\varphi = \Big(\frac{2}{\sqrt{3}}\Big)^{n+1} \frac{z}{D^n} (M_\varphi - M_r)^n, \qquad \eta_r = -\Big(\frac{2}{\sqrt{3}}\Big)^{n+1} \frac{z}{D^n} (M_\varphi - M_r)^n. \tag{17}$$

From (17) it follows that $\eta_\varphi + \eta_r = 0$ or, in view of (8),

$$\psi(r,t) = C(t)/r, \tag{18}$$

where $C(t)$ is an integration constant. In the following, instead of $C(t)$ it is expedient to consider the function $X(t)$ defined by the relation

$$[C(t)]^{1/n} = (C^0)^{1/n}[X(t)]^{-1} = \frac{B_1^{1/n}}{JJ_1} [M_r(b) - M_r(a) - \bar{Q}(b)][X(t)]^{-1}. \tag{19}$$

Here

$$J = 2\int\limits_0^{h/2} z^{(n+1)/n} dz, \qquad J_1 = \int\limits_a^b r^{-1-2/n} dr,$$

$M_r(a)$ and $M_r(b)$ are the values of $M_r(r)$ on the inner ($r = a$) and outer ($r = b$) radii of the plate. Next, let $\bar{Q}(r) = \int\limits_a^r Q(\zeta)\, d\zeta$; then, $\bar{Q}(b) = \int\limits_a^b Q(\zeta)\, d\zeta$. In this case, in view of (19), Eq. (18) becomes

$$\psi(r,t) = (C^0/r)[X(t)]^{-n} = \psi^0[X(t)]^{-n}. \tag{20}$$

From (20) it follows that the transition from $C(t)$ to $X(t)$ is performed on the basis of the hypothesis that the field $\psi(r,t)$ is similar to a certain stationary field $\psi^0(r)$. Moreover, the constant C^0 in (19) is chosen so that the field ψ^0 coincides with the field that follows from the solution of the examined problem under the assumption of steady-state creep of the material. Therefore, it is considered known and kinematically possible [1–3].

Using (18), from (8) we obtain the creep strain rate. Comparing them with (17), we have

$$M_\varphi - M_r = (\sqrt{3}/2)^{(n+1)/n}(C^0)^{1/n} D(r,t) r^{-2/n}[X(t)]^{-1}. \tag{21}$$

The values of $M_r(r)$ on the inner and outer radii of the plate are considered specified. Then, using (21), from the first equilibrium equation we obtain

$$M_r(r,t) = M_r(a) + \Big(\frac{\sqrt{3}}{2}\Big)^{(n+1)/n} (C^0)^{1/n}[X(t)]^{-1} \int\limits_a^r D(\zeta,t)\zeta^{-1-2/n}\, d\zeta + \bar{Q}(r). \tag{22}$$

From (22), taking into account the boundary condition for $r = b$ and (19), we have

$$\int\limits_a^b D(r,t) r^{-1-2/n}\, dr = \Big(\frac{2}{\sqrt{3}}\Big)^{(n+1)/n} \frac{JJ_1}{B_1^{1/n}} X(t).$$

In view of expressions (14) for $D(r,t)$ and (11) for a, we finally obtain

$$\int\limits_a^b \Big(2\int\limits_0^{h/2} [\phi(\mu)]^{1/n} z^{(n+1)/n}\, dz\Big) r^{-1-2/n}\, dr = JJ_1 X(t). \tag{23}$$

In the following, we need the expression $\sigma_i = lM_i$, where l is calculated according to (13) and M_i according to (21.) In view of expressions (19) for $(C^0)^{1/n}$, we obtain

$$lM_i = \frac{[\phi(\mu)]^{1/n}}{X(t)} \frac{M_r(b) - M_r(a) - \bar{Q}(b)}{JJ_1} r^{-2/n}|z|^{(1-n)/n} z. \tag{24}$$

Let us consider the function $t^0(r,z)$:

$$t^0 = [(\alpha+1)(m+1)B_2(\sigma_i^0)^{g+1}]^{-1}, \tag{25}$$

where the superscript 0 for the corresponding function indicates that it depends only on the coordinates of the points of the body. In other words, σ_i^0 is the expression of the stress intensity in the similar problem under the assumption of steady-state creep of the material, and $\sigma_i^0 = l^0 M_i^0$ [1]. Taking into account the expressions

$$l^0 = \frac{|z|^{(1-n)/n} z}{J}, \qquad M_i^0 = \frac{M_r(b) - M_r(a) - \bar{Q}(b)}{J_1}\, r^{-2/n}$$

we obtain

$$l^0 M_i^0 = \frac{M_r(b) - M_r(a) - \bar{Q}(b)}{J J_1}\, r^{-2/n} |z|^{(1-n)/n} z. \tag{26}$$

In view of (26), expression (24) is written as

$$l M_i = l^0 M_i^0 [\phi(\mu)]^{1/n} / X(t). \tag{27}$$

As the long-term strength criterion we use the Kats criterion. In this case, $\sigma_{*\,\mathrm{e}} = \sigma_i$. Substituting expression (27) into Eq. (2) and taking into account expression (1) for the function $\phi(\mu)$ and expressions (25), we obtain

$$\int\limits_1^{\mu} \Psi(z)\, dz = -[(m+1)t^0]^{-1} \int\limits_0^{t} [X(\tau)]^{-(g+1)}\, d\tau, \tag{28}$$

where $\Psi(z) = z^{m(n-g-1)/n} (1-z)^{-\alpha(g+1)/(n(\alpha+1))}$.

Equations (23) and (28) form a system of equations for the time function $X(t)$ and the damage parameter $\mu(r,z,t)$, from which it is possible to calculate the stress–strain state of the plate at any time before the beginning of fracture. Indeed, from (20) we obtain the relation $\psi(r,t) = \psi^0 [X(t)]^{-n}$, using which from (8) we find the creep strain rate field $\eta_\varphi(r,z,t)$, $\eta_r(r,z,t)$, the rate of deflection of the middle plane, and the deflection $w(r,\, t)$ for the known conditions of plate clamping, and from (9) we obtain the creep strain rate intensity $\eta_i(r,z,t)$. From (14), we calculate $D(r,t)$ and, thus, using (11) and (19), from (21) we find the intensity of the bending moments

$$M_i = M_\varphi - M_r = \frac{M_i^0}{aJ}\, \frac{D(r,t)}{X(t)}, \tag{29}$$

and from (22) the intensity of the radial bending moment $M_r(r,t)$ per unit length of the annular section of the plate. From (22) and (29), we obtain $M_\varphi(r,t) = M_r(r,t) + M_i(r,t)$. Using the known quantities M_i, M_φ, and M_r, from (27) we find the stress intensity $\sigma_i(r,z,t) = \sigma_i^0 [\phi(\mu)]^{1/n}/X(t)$, and from (13) the stress components $\sigma_\varphi(r,z,t)$ and $\sigma_r(r,z,t)$.

The time of onset of fracture is determined from the condition $\mu(r^*, z^*, t_*) = 0$. It is obvious that from the known function μ, it is possible to find not only t_* but also the coordinates r^* and z^* of the point at which μ vanishes for the first time.

It should be noted that depending on the characteristics of the material, the integral on the left side of (28) can represent the improper integral of the unbounded function at $\mu = 1$ (the lower limit of integration) and $\mu = 0$ (the upper limit of integration). For the convergence of the integral, one must impose restrictions on the characteristics of the material that are not hard and agree with available experimental data [5].

Let us consider some fundamental issues related to the solution of system (23), (28). It is obvious that this system generally admits only a numerical solution. Therefore, it is expedient to consider the case where $\alpha \neq 0$ and $m = 0$ in (28), which is typical of creep strengthening materials [2, 4], or the case of $\alpha = 0$ and $m \neq 0$ which is typical of many structural alloys [2, 4]. It is easy to show that for $\alpha \neq 0$ and $m = 0$, system (23), (28) has an analytical solution. In the case of $\alpha = 0$ and $m \neq 0$, from (28) we obtain

$$\mu^{m/n} = \Big(1 - \frac{\nu}{t^0} \int\limits_0^{t} X^{-(g+1)}\, d\tau\Big)^{\beta}, \tag{30}$$

where

$$\beta = \frac{m}{n + m(n-g-1)}, \qquad \nu = \frac{n + m(n-g-1)}{n(m+1)}. \tag{31}$$

According to (1), $\phi(\mu) = \mu^m$ for $\alpha = 0$ and $m \neq 0$; therefore, $\phi(\mu)^{1/n} = \mu^{m/n}$. Substitution of (30) into (23) yields

$$\int_a^b \Big[2\int_0^{h/2}\Big(1-\frac{\nu}{t^0}\int_0^t X^{-(g+1)}\,d\tau\Big)^{\beta} z^{(n+1)/n}\,dz\Big] r^{-1-2/n} dr = JJ_1 X(t). \tag{32}$$

The case of $\beta = 1$ is of practical interest. For $\beta = 1$, Eq. (32) becomes

$$1-\frac{\nu}{\bar{t}^0}\int_0^t [X(\tau)]^{-(g+1)}\,d\tau = X(t)$$

and has a solution

$$X(t) = (1 - t/\bar{t}^0)^{1/(g+2)}, \tag{33}$$

where the quantity $\bar{t}^0$ is obtained using the mean-value theorem

$$\int_a^b \Big(2\int_0^{h/2} [z^{(n+1)/n}/t^0(r,z)]\,dz\Big) r^{-1-2/n}\,dr = \frac{JJ_1}{\bar{t}^0}, \qquad \bar{t}^0 = t^0(\bar{r},\bar{z})$$

($\bar{r}$ and $\bar{z}$ are the coordinates of the "middle" point for $a \leqslant r \leqslant b$, and $0 \leqslant z \leqslant h/2$). Substitution of (33) into (30) yields

$$[\mu(r,z,t)]^{m/n} = \Big\{1 - \frac{t^0(\bar{r},\bar{z})}{t^0(r,z)}\Big[1-\Big(1-\frac{t}{\bar{t}^0}\Big)^{1/(g+2)}\Big]\Big\}. \tag{34}$$

Using (34), from the condition $\mu(r^*, z^*, t_*)$ we obtain

$$t_* = \bar{t}^0[1 - (1 - t_*^0/\bar{t}^0)^{g+2}]. \tag{35}$$

Here $t_*^0 = t^0(r^*, z^*)$ is the time of onset of fracture that corresponds to the solution of the problem in question under the assumption of steady-state creep of the material. For $\beta \neq 1$, Eq. (32) can be solved numerically using any of the well-known methods.

3. Before proceeding to an analysis of the solution that characterizes the stress–strain state of the plate at any time before the beginning of fracture, we write system (23), (28) in a different form in order to infer whether the procedure proposed in Secs. 1 and 2 is applicable to calculations of stress–strain states of other structural members (a pipe under internal pressure, a beam under bending, etc.). It is obvious that Eq. (28), which was obtained on the basis of Eq. (2), is valid for any structure. Let us consider Eq. (23). We first calculate the power of dissipated energy of the plate $W^0 = \eta_{ij}^0\sigma_{ij}^0$ under the assumption of steady-state creep of the plate material. It is known that $\eta_{ij}^0\sigma_{ij}^0 = \eta_i^0\sigma_i^0 = B_1(\sigma_i^0)^n\sigma_i^0$ [2]. Taking into account that $\sigma_i^0 = l^0 M_i^0$ and using Eq. (26), we obtain

$$W^0 = B_1\Big(\frac{M_r(b) - M_r(a) - \bar{Q}(b)}{JJ_1}\Big)^{n+1} z^{(n+1)/n} r^{-2(n+1)/n}.$$

The power of dissipated energy of the entire plate is

$$\int_V W^0\,dV = A\int_0^{2\pi} d\varphi \int_a^b \Big(2\int_0^{h/2} z^{(n+1)/n}\,dz\Big) r^{-1-2/n}\,dr = 2\pi A JJ_1. \tag{36}$$

The coefficient $A = B_1[(M_r(b) - M_r(a) - \bar{Q}(b))/(JJ_1)]^{n+1}$. With the use of (36), Eq. (23) is written as

$$\int_V W^0[\phi(\mu)]^{1/n}\,dV = X(t)\int_V W^0\,dV. \tag{37}$$

It is obvious that Eq. (37) is valid not only for plates loaded by a bending moment or pressure but also for other structural members under various loading and clamping conditions; in this case, W^0 is the power of dissipated energy of the structural member considered and V is its volume.

Let us analyze the solution obtained. As noted above, the creep strain rate field and intensity are found from (8) and (9) using (18) and (19): $\eta_\varphi(r,z,t) = \eta_\varphi^0[X(t)]^{-n}$, $\eta_r(r,z,t) = \eta_r^0[X(t)]^{-n}$, $\eta_i(r,z,t) = \eta_i^0[X(t)]^{-n}$, where $\eta_\varphi^0 = z\psi^0/r$ and $\eta_r^0 = z\,d\psi^0/dr$. For the constant C^0 and the expression $\psi^0 = C^0/r$, it follows from (19) that the quantities η_φ^0, η_r^0, and η_i^0 coincide with the same quantities calculated in the steady-state creep problem [1–3].

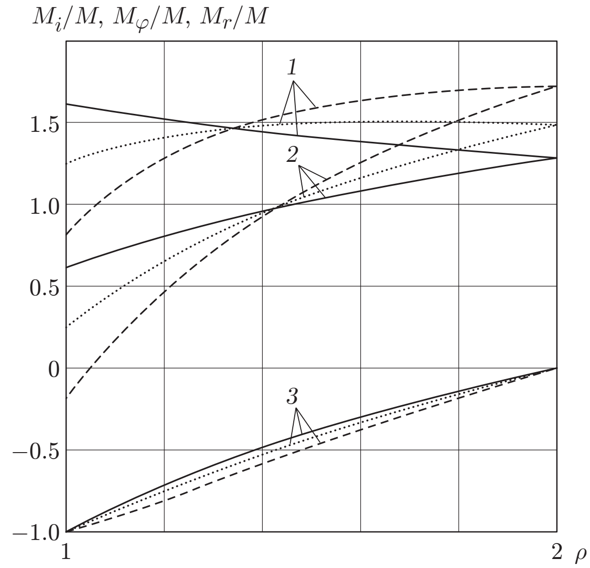

Fig. 2. Curves $M_i/M(\rho)$ (1), $M_\varphi/M(\rho)$ (2), and $M_r/M(\rho)$ (3) for $\varphi = \text{const}$ and $\tau_0 = 0$ (solid curves), $\bar\tau = 2.1$ (dotted curves), and $\tau_* \approx 2.456$ (dashed curves).

Therefore, the quantities η^0_φ, η^0_r, and η^0_i are considered known, and they are kinematically possible. The given creep strain rate field is related to the corresponds stress field by the steady-state creep law. The indicated stress field may not satisfy the equilibrium equations. In the case considered, the stress field is the field defined by relations (10), which, with the use of (11), (18), and (19), can be written as

$$\sigma_\varphi(r,z,t) = \sigma^0_\varphi[\phi(\mu)]^{1/n}/X(t), \qquad \sigma_r(r,z,t) = \sigma^0_r[\phi(\mu)]^{1/n}/X(t). \tag{38}$$

Here σ^0_φ and σ^0_r are the stress components that correspond to the solution of the steady-state creep problem [1–3]; hence, they are statically admissible. The stress components in (38) are not statically admissible because they do not satisfy the equilibrium equations. The stresses σ_φ and σ_r defined by relations (13) are statically admissible; in (13), M_φ and M_r satisfy the equilibrium equations (15) [M_r is found from Eq. (22), and M_φ from Eq. (21) with the use of Eq. (22)]. Taking into account that $\sigma_i = lM_i$ and using (26), from (24) we obtain

$$\sigma_i(r,z,t) = \sigma^0_i[\phi(\mu)]^{1/n}/X(t), \tag{39}$$

where σ^0_i is the stress intensity in the steady-state creep problem.

We note that substitution of (38) into the expression for the stress intensity (4) yields relation (39). From this it follows that the field (38) can be rendered statically admissible. We set

$$\sigma_\varphi(r,z,t) = \sigma^0_\varphi[\phi(\mu)]^{1/n}/X(t) + C, \qquad \sigma_r(r,z,t) = \sigma^0_r[\phi(\mu)]^{1/n}/X(t) + C. \tag{40}$$

The hydrostatic component C is calculated from the condition that the stress components (40) satisfy the equilibrium equations. The equation for the quantity C is easy to obtain and, hence, is not given here.

The aforesaid leads to the following conclusion. The solution of the problem of the stress–strain state of ring and circular plates with allowance for damage accumulation in the material during creep can be reduced to solving the same problem under the assumption of steady-state creep. Methods for the solution of such problems are fairly well elaborated; therefore, the solution of the steady-state creep problem is considered known [1–3]. To obtain the unsteady creep solution, it is necessary to multiply the known solution of the steady-state creep by the functions $\mu(r,z,t)$ and $X(t)$ that are the solution of system (28), (37).

This conclusion remains valid for other structural members: thick-walled pipes loaded by internal pressure, bendable beams, rotating disks, etc. This implies that the result obtained can be extended to an arbitrary nonuniformly heated body, the more so as system (28), (37) is valid for exactly those bodies.

4. Figure 2–5 gives calculation results for a ring plate loaded by a bending moment $M_r(a) = -M = -600$ N·m on the inner radius and by $M_r(b) = 0$ on the outer radius. The plate thickness is $h = 0.01$ m,

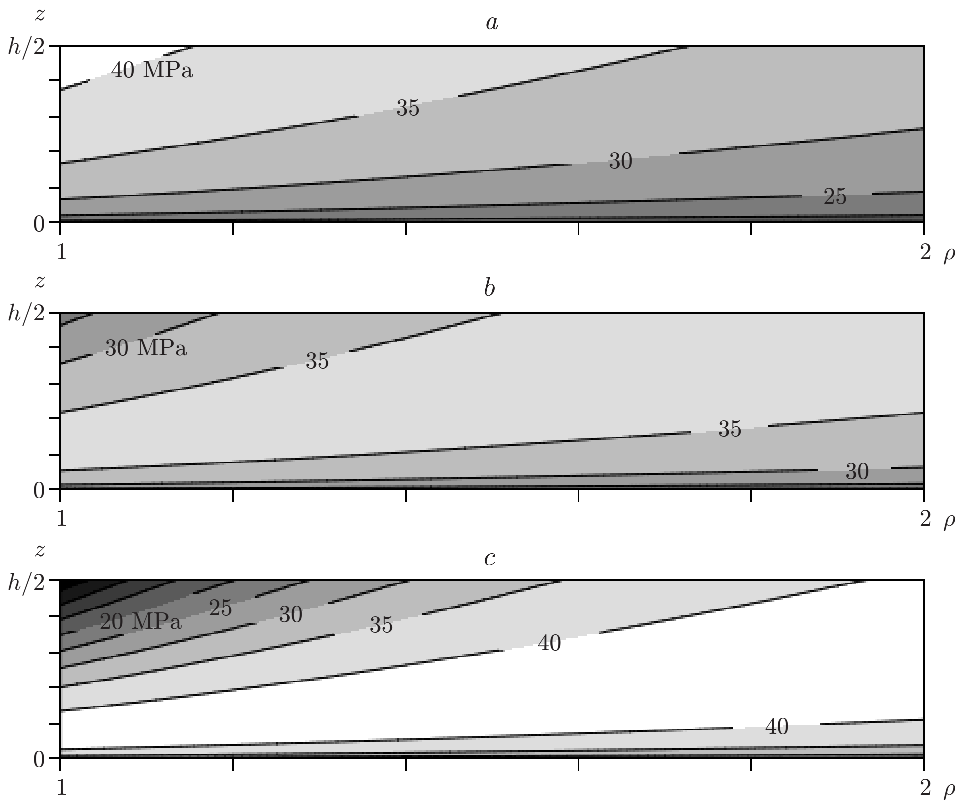

Fig. 3. Isolines of the stress intensity σ_i in section $\varphi = \text{const}$, $1 \leqslant \rho \leqslant 2$, $0 \leqslant z \leqslant h/2$ for $\tau_0 = 0$ (a), $\bar{\tau} = 2.1$ (b), and $\tau_* \approx 2.456$ (c).

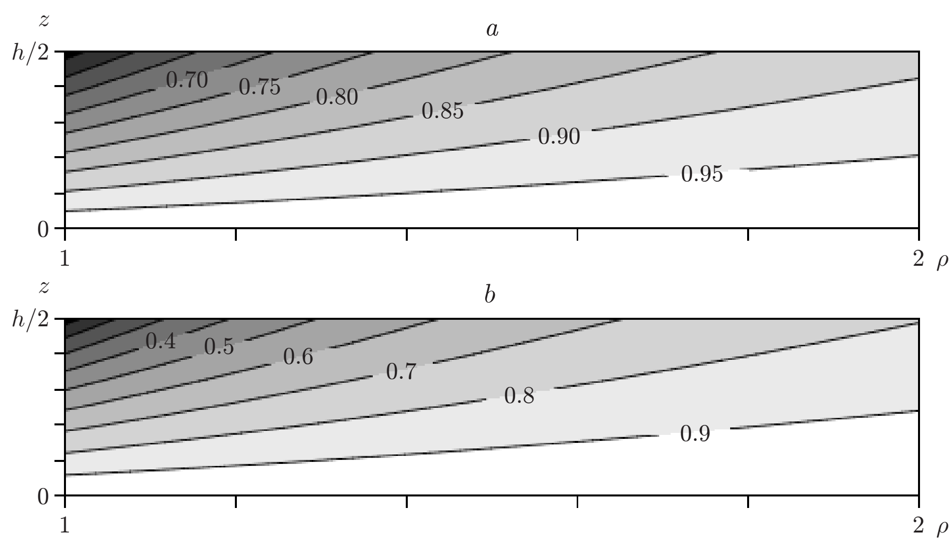

Fig. 4. Isolines of the damage function $\mu(\rho, z, \tau)$ for $\bar{\tau} = 2.1$ (a) and $\tau_* \approx 2.456$ (b).

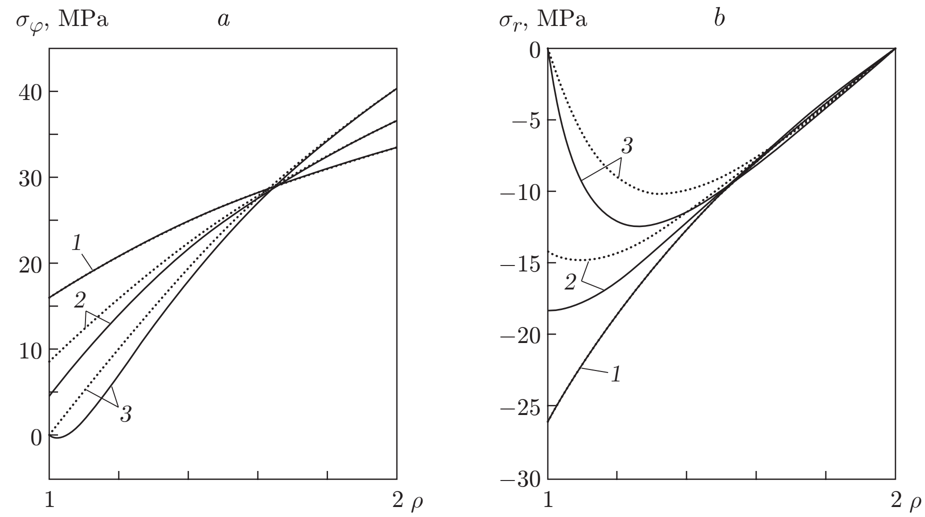

Fig. 5. Curves $\sigma_\varphi(\rho)$ (a) and $\sigma_r(\rho)$ (b) at $z = h/2$ and $\tau_0 = 0$ (1), $\bar{\tau} = 2.1$ (2), and $\tau_* \approx 2.456$ (3); the solid curves are calculated using formulas (13), (21), and (22); the dotted curves are calculated using formulas (38).

the radii are $a = 0.05$ m and $b = 0.1$ m, and the dimensionless radius $\rho = b/a$ varies in the range $1 \leqslant \rho \leqslant 2$. On the outer contour ($\rho = 2$), the plate is hinge supported, and the inner contour ($\rho = 1$) is not fixed. The calculations were performed for the following material characteristics: $\alpha = 0$, $n = 6$, $g = 4.75$, $m = 8$, $B_1 = 3.5172 \cdot 10^{-15}$ MPa^{-n}/h, and $B_2 = 2.7563 \cdot 10^{-15}$ MPa^{-g-1}/h. These characteristics correspond to D16 alloy at $T = 250$°C [4.] In the case considered, $\beta = 1$ [according to (31)]; therefore, the functions $X(t)$ and $\mu(r, z, t)$ and the time of onset of fracture t_* are found from (33)–(35).

In Fig. 2, curves 1–3 show the distributions of the functions M_i/M, M_φ/M, and M_r/M, respectively, versus ρ in the cross sections of the plate $\varphi =$ const. The calculation was performed in accordance with Eqs. (21) and (22). The solid, dotted, and dashed curves correspond to the following three fixed values of the time $\tau = t/t_*^0$: the initial value $\tau_0 = 0$, the intermediate value $\bar{\tau} = 2.1$, and the time of onset of fracture $\tau_* = t_*/t_*^0 \approx 2.456$ ($t_*^0 = 1.867 \cdot 10^4$ h). It is obvious that during creep in the plate there is a continuous redistribution of the bending moments, which cannot be ignored even in the stage of design. The bending moments M_φ and M_r and the intensity M_i differ significantly from M_φ^0, M_r^0, and M_i^0 (solid curves in Fig. 2) obtained from the solution of the steady-state creep problem.

Figure 3 shows isolines of the stress intensity $\sigma_i = lM_i$ in the section $\varphi =$ const, $1 \leqslant \rho \leqslant 2$, $0 \leqslant z \leqslant h/2$ for the same times as in Fig. 2. The quantity lM_i is calculated by formula (24) or (27). Figure 4 shows isolines of the damage function $\mu(\rho, z, \tau)$ (34) at the times $\bar{\tau}$ and τ_*. At $\tau = \tau_0$, we have $\mu = 1$ over the entire section. The isolines presented in Figs. 3 and 4 provide evidence for damage accumulation in the plate material at various sections. It is obvious that the regions $0 \leqslant \varphi \leqslant 2\pi$, $\rho^* = 1$, $z^* = \pm h/2$ are dangerous. From exactly these regions, the fracture front begins to propagate at $\tau_* > t_*/t_*^0$. In the case considered, the value of t_* exceeds t_*^0 by more than a factor of 2.4. This indicates that t_*^0 is a lower-bound estimate of the time t_*, i.e., is an approximate time of onset of fracture of the plate.

In Fig. 5, the solid curves 1–3 show the stresses σ_φ and σ_r versus ρ at $z = h/2$ for the same times as in Fig. 2. The calculation was performed with the use of relations (13), (21) and (22). The dashed curves show the dependences of the stress components σ_φ and σ_r calculated according to (38). From an analysis of the data in Fig. 5, it follows that the stress field (38) can be used as an estimate.

In Fig. 6, the time of onset of fracture t_* versus the bending moment M applied to the inner radius of the plate is plotted in logarithmic coordinates. It is evident that the dependence is a straight line. Using this straight line or its continuation, it is possible to predict the time of onset of fracture for various values of the bending moment M. The point A in Fig. 6 corresponds to $M = 350$ N $\cdot$ m and $t_* = 1.017 \cdot 10^6$ h.

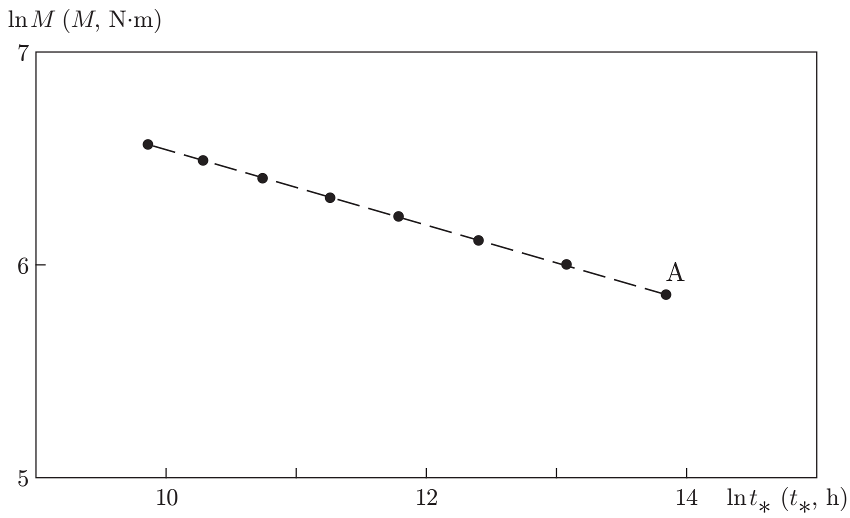

Fig. 6. The time of onset of fracture t_* versus the bending moment M applied to the inner radius of the plate.

In conclusion, it should be noted that the procedure for calculating the stress–strain state and damage accumulation for ring and circular plates under different loading and clamping conditions (for example, bending by a moment applied to the outer radius, loading by a uniformly distributed pressure, etc.) is similar to the proposed in the present paper. Some examples of solutions of these problems under the assumption of steady-state creep are considered in [1]. To obtain a solution that takes into account damage accumulation, it is necessary to multiply the known steady-state creep solution by the functions of the coordinates and time that are found from system (28), (37). This system of equations, as shown in Sec. 3, can be used to calculate not only plates but also other structural members with various loading and clamping conditions.

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